International Journal of Theoretical Physics, Vol. 10, No. 2 (1974), pp. 123–143

Imprecision Description of the Threshold Behaviour of the High-Energy Pairs Annihilation and Production Processes

E. PAPP

Polytechnic Institute of Cluj, Physics Department, Cluj, Romania

Received: 20 *December* 1973

Abstract

The time operators corresponding to the whole Dirac and Klein–Gordon fields and the expressions of the time imprecisions associated to the pairs annihilation and subsequent production processes have been evaluated. In these conditions there exists the possibility of assuming the existence of a threshold behaviour of the above processes. Certain peculiarities of the threshold behaviour obtained in this way are related with certain results obtained by virtue of the imprecision description of the coulombian interaction. Finally, the meaning and role of the electromagnetic natural space unit is also discussed.

1. *Introduction*

In line with previous papers (Papp, 1973a, b), we have to consider that the quanto-mechanical description of time measurements implies the existence of a binary entity whose imaginary part expresses, in a 'natural' way (Bohm *et al.*, 1970), the objective imprecision bound of the measurements performed. This binary entity expresses, in fact, the extension of the usual hermitic observable in the conditions of which the presence and role of the measuring apparatus are considered. As a consequence, the possibility exists of defining certain high-energy structural effects implied by the existence of natural space and time units as lower bounds of the corresponding imprecisions (Papp, 1973a).

There also exists the possibility of introducing into the mathematical formalism of quantum mechanics, or of quantum field theory, certain parameters possessing—directly or indirectly—the meaning of discrete-space or discrete-time quanta. In this sense non-local field theory, the non-definite metric method, the Lagrangians with higher derivatives, and non-linear field theory (see, e.g., Vialtzew, 1965) are mutually correlated methods which imply or support, in one way or another, the existence of a discrete space. When performing the binary space-time description within quantum theories implying or supporting the existence of a discrete space-time, agreement between the existence of discrete space-time quanta and the existence of the space-time imprecisions is needed. In these conditions results may be obtained which express peculiarities of a unified quanto-mechanical description of both space-time and matter.

Continuing the imprecision description in previous papers (Papp, 1973a, b), the time operators of the Dirac (D) and Klein–Gordon (K–G) fields are evaluated in Section 2. The behaviour of the above operators under time-inversion is analysed in Section 3. Section 4 defines the proper-time operators of the D and K–G fields. The meaning and form of the time- and proper-time imprecisions is discussed in Section 5. In these conditions one proves the existence of a time-imprecision contribution associated with the pairs annihilation process. The threshold velocities implied by the existence of the above time- and proper-time imprecisions are also calculated. Section 6 considers resonance-emission approximation of the so-implied high-energy particle production processes. Explanations of certain particle production processes are proposed.

In close connection with the above results the quanto-mechanical peculiarities of the coulombian interaction are analysed in Section 7. A particular connection between the threshold behaviour implied by the time-imprecision description and the threshold behaviour implied by the imprecision description of the electrostatic potential energy may be established. In this respect the existence of a certain effective discrete-space structure is needed. Finally, the definition of the natural 'electromagnetic' space unit is accomplished in Section 8.

Throughout this paper the collision processes between the charged particle and the charged anti-particle are considered in the centre-of-mass system. We shall assume that the particle and the corresponding anti-particle possess the same rest-mass. The interaction representation is also used.

2. *Time Operators of the D and K–G Fields*

We shall define the time operator corresponding to the D field of the free evolution along the axis Ox_1 and to a given value of the spin by the relation

$$\mathscr{T}_1^{(s)}(t) = \int dx_1 \psi_1^{(s)*}(x_1, t) x_1 \hat{v}_1^{-1} \psi_1^{(s)}(x_1, t) \tag{2.1}$$

where

$$\psi_1^{(s)}(x_1, t) = (2\pi)^{-1/2} \int dp_1 \sqrt{\left(\frac{m_0}{p_0}\right)} [b(p_1, s)u(p_1, s) \exp ipx + d^*(p_1, s)v(p_1, s) \exp(-ipx)] \tag{2.2}$$

represents the whole operator of the D-field operator, $px = p_1 x_1 - p_0 t$, $p_0 = (p_1^2 + m_0^2)^{1/2}$ and where, as previously (Papp, 1973b), $\hat{v}_1^{-1}$ expresses the inverse-velocity operator.

Using the spinorial relations

$$u^*(p_1, s')u(p_1, s) = v_1^*(p_1, s')v(p_1, s) = \frac{p_0}{m_0}\,\delta_{s's} \tag{2.3}$$

$$u^*(-p_1, s')v(p_1, s) = v^*(-p_1, s')u(p_1, s) = 0 \tag{2.4}$$

$$v^*(p_1, s')\frac{\partial}{\partial p_1}v(p_1, s) = u^*(p_1, s')\frac{\partial}{\partial p_1}u(p_1, s) = \frac{p_1}{2m_0p_0}\,\delta_{s's} \tag{2.5}$$

$$u^*(-p_1, s')\frac{\partial}{\partial p_1}v(p_1, s) = v^*(-p_1, s')\frac{\partial}{\partial p_1}u(p_1, s) = \frac{1}{2p_0}\,\delta_{s's} \tag{2.6}$$

where the relations (2.5)-(2.6) maintain theirs form only in the one-dimensional case, one obtains

$$\mathscr{T}_1^{(s)}(t) = T_1^{(s)}(t) - \bar{T}_1^{(s)}(t) + \tilde{I}_1^{(s)} - \tilde{I}_1^{(s)*} \tag{2.7}$$

as soon as the D-field production and annihilation operators obey, in the weak sense, the boundary conditions

$$\lim_{p_1\to 0} p_1^{-1/2}b(p_1, s) = 0, \qquad \lim_{p_1\to\infty} b(p_1, s) = 0 \tag{2.8}$$

$$\lim_{p_1\to 0} p_1^{-1/2}d(p_1, s) = 0, \qquad \lim_{p_1\to\infty} d(p_1, s) = 0 \tag{2.9}$$

In these conditions

$$T_1^{(s)}(t) = \int dp_1 b^*(p_1, s)\left[i\frac{p_0}{p_1}\frac{\partial}{\partial p_1} + t - i\frac{{m_0}^2}{{p_1}^2p_0}\right]b(p_1, s) \tag{2.10}$$

expresses the binary time operator of the D particles,

$$\bar{T}_1^{(s)}(t) = \int dp_1 d^*(p_1, s)\left[i\frac{p_0}{p_1}\frac{\partial}{\partial p_1} + t\right]d(p_1, s) \tag{2.11}$$

expresses the binary time-operator of the D anti-particles, and

$$\tilde{I}_1^{(s)} = i\int dp_1\,\frac{m_0}{2p_1p_0}\,b^*(p_1, s)d^*(-p_1, s) \tag{2.12}$$

represents a time-imprecision operator whose meaning will be subsequently analysed. We may thus conclude that by virtue of relations (2.5)-(2.6), the time operator of the D field (corresponding to an arbitrary direction) may be adequately defined only in the one-dimensional case.

Defining similarly the time operator of the one-dimensional K–G field by the relation

$$\mathscr{T}_1(t) = i\int dx_1\Phi^*(x_1, t)\overleftrightarrow{\partial_t}x_1\hat{v}_1^{-1}\Phi(x_1, t) \tag{2.13}$$

where

$$\Phi(x_1, t) = (2\pi)^{-1/2} \int \frac{dp_1}{\sqrt{(2p_0)}} [a(p_1) \exp ipx + b^*(p_1) \exp(-ipx)] \quad (2.14)$$

expresses the field operator of the whole K–G field, one obtains

$$\mathscr{T}_1(t) = T_1(t) - \bar{T}_1(t) + \tilde{I}_1 - \tilde{I}_1^* \quad (2.15)$$

where

$$T_1(t) = \int dp_1 a^*(p_1) \left[i\frac{p_0}{p_1}\frac{\partial}{\partial p_1} + t - i\frac{m_0^2}{p_1^2 p_0} \right] a(p_1) \quad (2.16)$$

and

$$\bar{T}_1(t) = \int dp_1 b^*(p_1) \left[i\frac{p_0}{p_1}\frac{\partial}{\partial p_1} + t \right] b(p_1) \quad (2.17)$$

represent the previously defined time operator of the K–G particle field and the time operator of the K–G anti-particle field, respectively, and where

$$\tilde{I}_1 = i \int dp_1 \frac{1}{2p_0} a^*(p_1) b^*(-p_1) \quad (2.18)$$

expresses a certain time-imprecision operator. In agreement with the general requirements to perform, along an arbitrary direction a consistent field-theoretical time description (Papp, 1973b), we have restricted ourself to the one-dimensional case. It has been assumed that the boundary conditions needed are fulfilled.

3. *Behaviour of the Time Operators under Time-Inversion*

In agreement with the usual results (Bjorken & Drell, 1965) the annihilation operators $b(\mathbf{p}, s)$ and $d(\mathbf{p}, s)$ transform under time-inversion as

$$\mathscr{U} b(\mathbf{p}, s) \mathscr{U}^{-1} = -b(-\mathbf{p}, -s) \exp i\alpha_+(\mathbf{p}, s) \quad (3.1)$$

$$\mathscr{U} d(\mathbf{p}, s) \mathscr{U}^{-1} = -d(-\mathbf{p}, -s) \exp [-i\alpha_-(\mathbf{p}, s)] \quad (3.2)$$

where $\mathscr{U}$ expresses the unitary operator of the time-inversion and where $\alpha_+(\mathbf{p}, s)$ and $\alpha_-(\mathbf{p}, s)$ are phase functions corresponding to the particle and anti-particle respectively.

But it may be proved that

$$Tu(\mathbf{p}, \pm s) = \exp i\alpha_+(\mathbf{p}, s) u^*(-\mathbf{p}, \mp s) = \mp u^*(-\mathbf{p}, \mp s) \quad (3.3)$$

$$Tv(\mathbf{p}, \pm s) = \exp i\alpha_-(\mathbf{p}, s) v^*(-\mathbf{p}, \mp s) = \pm v^*(-\mathbf{p}, \mp s) \quad (3.4)$$

where $T = i\gamma_1\gamma_3$ is the quanto-mechanical operator of the time-inversion operation. Changing the sign of the momentum $\mathbf{p}$, relations (3.3) and (3.4) remain valid. As a consequence we may neglect the momentum dependence of the α_+ and α_- phase functions. In these conditions we have

$$\mathscr{J} T_1^{(s)}(t) \mathscr{J}^{-1} = -T_1^{(-s)}(-t), \qquad \mathscr{J} \bar{T}_1^{(s)}(t) \mathscr{J}^{-1} = -\bar{T}_1^{(-s)}(-t) \tag{3.5}$$

$$\mathscr{J} \tilde{I}_1^{(s)} \mathscr{J}^{-1} = -\tilde{I}_1^{(-s)}, \qquad \mathscr{J} \tilde{I}_1^{(s)*} \mathscr{J}^{-1} = -\tilde{I}_1^{(-s)*} \tag{3.6}$$

so that the behaviour of the whole time operator under time-inversion is given by

$$\mathscr{J} \mathscr{T}_1^{(s)}(t) \mathscr{J}^{-1} = -\mathscr{T}_1^{(-s)}(-t) \tag{3.7}$$

where $\mathscr{J} = \mathscr{U}\mathscr{K}$ and where $\mathscr{K}$ expresses the complex conjugation operator of the c-numbers. According to relations (3.3) and (3.4) the equality

$$\exp i[\alpha_+(s) \pm \alpha_-(s)] = -1 \tag{3.8}$$

has been used. The sign changes of the spin variable and time operator so implied agree with the usual peculiarities of the time-inversion operation. In this respect relations (3.5)-(3.7) may also be considered as testifying relations of any time operator associated with a free field. If we consider the existence of a momentum dependence of the α_+ and α_- phase functions, like relations (3.1) and (3.2), the appearance under time-inversion of certain additional time-shift contributions would be implied. Thus

$$\mathscr{J} \mathscr{T}_1^{(0)}(t) \mathscr{J}^{-1} = -\mathscr{T}_1^{(0)}(-t) - \sum_{\pm s} \int dp_1 b^*(p_1, s) b(p_1, s) \frac{p_0}{p_1} \frac{\partial}{\partial p_1} \alpha_+(p_1, s)$$
$$- \sum_{\pm s} \int dp_1 d^*(p_1, s) d(p_1, s) \frac{p_0}{p_1} \frac{\partial}{\partial p_1} \alpha_-(p_1, s) \tag{3.9}$$

where $\mathscr{T}_1^{(0)}(t) = \mathscr{T}_1^{(s)}(t) + \mathscr{T}_1^{(-s)}(t)$.

Commencing with relations

$$\mathscr{U} a(p_1) \mathscr{U}^{-1} = a(-p_1) \tag{3.10}$$

$$\mathscr{U} b(p_1) \mathscr{U}^{-1} = b(-p_1) \tag{3.11}$$

which define the time-inversion operation of the free K–G field, it may be proved—excluding the presence of the spin—that the behaviour of the K–G time-operator under time-inversion is similar to that of the D field.

4. *Proper-Time Operators of the D and K–G Fields*

The proper-time operator of the K–G field corresponding to the quanto-mechanical proper time

$$\hat{s} = x_\mu \hat{p}_\mu (\hat{p}_\nu \hat{p}_\nu)^{-1/2} \tag{4.1}$$

where $\hat{p}_\mu = -i(\partial/\partial x_\mu)$, which is identical, up to the sign, with the one previously used (Papp, 1972), may be defined as

$$\mathcal{S}_3(t) = i \int d\mathbf{x} \Phi_3^*(\mathbf{x}, t) \overleftrightarrow{\partial}_t \hat{s} \Phi_3(\mathbf{x}, t) \tag{4.2}$$

where the three-dimensional case has been considered. Performing the calculations it results

$$\mathcal{S}_3(t) = S_3(t) - \bar{S}_3(t) + \tilde{\tilde{I}}_3 - \tilde{\tilde{I}}_3^* \tag{4.3}$$

where

$$S_3(t) = \int d\mathbf{p} a^*(\mathbf{p}) \left[i \frac{\mathbf{p}}{m_0} \cdot \frac{\partial}{\partial \mathbf{p}} - m_0 \frac{t}{p_0} + \frac{5i}{2m_0} \right] a(\mathbf{p}) \tag{4.4}$$

is the proper-time operator of the K–G particle field,

$$\bar{S}_3(t) = \int d\mathbf{p} b^*(\mathbf{p}) \left[i \frac{\mathbf{p}}{m_0} \cdot \frac{\partial}{\partial \mathbf{p}} - m_0 \frac{t}{p_0} + \frac{i}{2m_0} \right] b(\mathbf{p}) \tag{4.5}$$

is the proper-time operator of the K–G anti-particle field, and where

$$I_3 = -\frac{i}{2m_0} \int d\mathbf{p} a^*(-\mathbf{p}) b^*(\mathbf{p}) \tag{4.6}$$

expresses a certain proper-time imprecision operator. In order to obtain the above results the use of the boundary condition referring to the vanishing of the creation and annihilation operators at the origin is not implied. The calculations may be similarly performed in the two- and one-dimensional cases. It may be proved that under time-inversion

$$\mathcal{J} \mathcal{S}_j(t) \mathcal{J}^{-1} = -\mathcal{S}_j(-t), \qquad j = 1, 2, 3 \tag{4.7}$$

and this behaviour agrees with that obtained for the time operators.

In contrast to the above results, the definition of the D field proper-time operator needs wider discussion. In this sense, if we try to define, irrespective of the spin-value, the proper-time operator of the three-dimensional D field as

$$\mathcal{S}_3^{(0)}(t) = \int d\mathbf{x} \psi_3^{(0)*}(\mathbf{x}, t) \hat{s} \psi_3^{(0)}(\mathbf{x}, t) \tag{4.8}$$

there would arise, as a consequence of the relations

$$v^*(-\mathbf{p}, -s)\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} u(\mathbf{p}, s) = \frac{p_3}{2p_0} \tag{4.9}$$

$$u^*(-\mathbf{p}, -s)\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} v(\mathbf{p}, s) = \frac{p_3}{2p_0} \tag{4.10}$$

interference contributions in respect of the spin variable. Consequently, the proper-time operator, corresponding to a well-defined spin value, cannot generally be defined in the three-dimensional case. Performing the calculations in the two-dimensional case, when

$$u^*(-\mathbf{p}, s')\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} v(\mathbf{p}, s) = \frac{p_1 - ip_2}{2p_0} \delta_{s's} \tag{4.11}$$

$$v^*(-\mathbf{p}, s')\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} u(\mathbf{p}, s) = \frac{p_1 + ip_2}{2p_0} \delta_{s's} \tag{4.12}$$

and using the relations

$$u^*(\mathbf{p}, s')\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} u(\mathbf{p}, s) = v^*(\mathbf{p}, s')\mathbf{p} \cdot \frac{\partial}{\partial \mathbf{p}} v(\mathbf{p}, s) = \frac{\mathbf{p}^2}{2m_0 p_0} \delta_{s's} \tag{4.13}$$

which are valid irrespective of the dimension, one obtains

$$\begin{aligned} \mathscr{S}_2^{(s)}(t) = & \int dp_1 dp_2 b^*(p_1, p_2, s)\left[i \frac{p_1}{m_0} \frac{\partial}{\partial p_1} + i \frac{p_2}{m_0} \frac{\partial}{\partial p_2} - t \frac{m_0}{p_0} + \frac{2i}{m_0}\right] b(p_1, p_2, s) \\ & - \int dp_1 dp_2 d^*(p_1, p_2, s)\left[i \frac{p_1}{m_0} \frac{\partial}{\partial p_1} + i \frac{p_2}{m_0} \frac{\partial}{\partial p_2} - t \frac{m_0}{p_0}\right] d(p_1, p_2, s) \\ & + i \int dp_1 dp_2 \frac{p_1 - ip_2}{2p_0^2} d(-p_1, -p_2, s) b(p_1, p_2, s) \\ & - i \int dp_1 dp_2 \frac{p_1 + ip_2}{2p_0^2} b^*(-p_1, -p_2, s) d^*(p_1, p_2, s) \end{aligned} \tag{4.14}$$

where a well-defined value of the spin has been considered. Consequently

$$\begin{aligned} \mathscr{J}\mathscr{S}_2^{(s)}(t)\mathscr{J}^{-1} = & -\mathscr{S}_2^{(-s)}(-t) + 2 \int dp_1 dp_2 \frac{p_2}{2p_0^2} d(-p_1, -p_2, s) b(p_1, p_2, -s) \\ & + 2 \int dp_1 dp_2 \frac{p_2}{2p_0^2} b^*(-p_1, -p_2, -s) d^*(p_1, p_2, -s) \end{aligned} \tag{4.15}$$

so that the two-dimensional proper-time operator rigorously fulfills the requirements needed by the time-inversion only in the one-dimensional case, or approximately in the two-dimensional case when the presence of the last two terms in expression (4.15) may be neglected. In this respect there arises a certain difference between the proper-time operators of the D and K-G fields.

5. *Time-Imprecision Contributions of the Time and Proper-Time Operators*

In order to obtain the time imprecisions let us define the free particle-antiparticle states of the D and K-G fields as

$$|+, -, s\rangle = \int dp_1 dp_2 f(p_1, s)\bar{f}(p_2, s) b^*(p_1, s) d^*(p_2, s)|0\rangle \tag{5.1}$$

and

$$|+,-\rangle = \int dp_1 dp_2 g(p_1)\bar{g}(p_2) a^*(p_1) b^*(p_2)|0\rangle \tag{5.2}$$

respectively. The functions $f(p_1, s)$ and $g(p_1)$ denote, in the momentum representation, the particle amplitudes, whereas $\bar{f}(p_1, s)$ and $\bar{g}(p_1)$ are the anti-particle amplitudes.

In the coordinate representation the particle amplitudes take the form

$$\psi^{(+)}(x_1, t_1, s) = (2\pi)^{-1/2} \int dp_1 \sqrt{\left(\frac{m_0}{p_0}\right)} u(p_1, s) f(p_1, s) \exp ipx \tag{5.3}$$

$$\varphi^{(+)}(x_1, t) = (2\pi)^{-1/2} \int dp_1 \frac{1}{\sqrt{(2p_0)}} g(p_1) \exp ipx \tag{5.4}$$

The anti-particle amplitudes are given by

$$\bar{\psi}^{(+)}(x_1, t, s) = (2\pi)^{-1/2} \int dp_1 \sqrt{\left(\frac{m_0}{p_0}\right)} u(p_1, s) \bar{f}(p_1, s) \exp ipx \tag{5.5}$$

and

$$\bar{\varphi}^{(+)}(x_1, t) = (2\pi)^{-1/2} \int dp_1 \frac{1}{\sqrt{(2p_0)}} \bar{g}(p_1) \exp ipx \tag{5.6}$$

respectively. We shall consider that the particle amplitudes $f(p_1, s)$ and $g(p_1)$ take appreciable values only around a certain positive momentum average $\langle p_1 \rangle$.

The one-dimensional free D equation possesses, besides the positive energy solution $\psi^{(+)}(x_1, t, s)$, the solution $\gamma_0 \psi^{(+)*}(x_1, -t, s)$ obtained by time-inversion. In these conditions we may consider that the anti-particle amplitude $\bar{\psi}^{(+)}(x_1, t, s)$ has to be defined by the above-mentioned time-reversed solution:

$$\bar{\psi}^{(+)}(x_1, t, s) = \gamma_0 \psi^{(+)*}(x_1, -t, s) \tag{5.7}$$

so that

$$\bar{f}(p_1, s) = f^*(-p_1, s) \tag{5.8}$$

In respect of the particle–anti-particle elastic collision processes we may consider that the functions $\psi^{(+)}(x_1, t, s)$ and $\bar{\psi}^{(+)}(x_1, t, s)$ describe, in the centre-of-mass system, the incoming ($t \ll 0$) evolution of the free particle and anti-particle respectively. Assuming that the particle–anti-particle interaction is described by a S-matrix, or by an interaction Hamiltonian, which is invariant under time-inversion, it may be proved that relation (5.8) also maintains its validity in respect with the outgoing ($t \gg 0$) states. Indeed, denoting with $\psi^{(+)\mathrm{out}}(x_1, t, s)$ and $\bar{\psi}^{(+)\mathrm{out}}(x_1, t, s)$ the outgoing particle and anti-particle amplitude, respectively, the equality

$$\bar{\psi}^{(+)\mathrm{out}}(x_1, t, s) = \gamma_0 \psi^{(+)\mathrm{out}*}(x_1, -t, s) \tag{5.9}$$

becomes

$$\bar{f}^{\text{out}}(p_1, s) \equiv \bar{f}(p_1, s) \exp 2i\delta(p_1) = f^{\text{out}*}(-p_1, s) \tag{5.10}$$

where—in agreement with the usual results of collision theory—the phase-shift $\delta(p_1)$ is an odd function in respect to the momentum and where $f^{\text{out}}(p_1, s)$ and $\bar{f}^{\text{out}}(p_1, s)$ are the particle and anti-particle amplitudes in the momentum representation, respectively. Thus, up to the phase factor $\exp 2i\delta(p_1)$, equalities (5.8) and (5.10) are identical. As one would expect, the anti-particle amplitude may be also obtained by the charge-conjugation transformation

$$\bar{\psi}^{(+)}(x_1, t, s) = C\gamma_0\psi^{(-)*}(x_1, t, s), \qquad C = i\gamma_2\gamma_0 \tag{5.11}$$

of the negative energy solution

$$\psi^{(-)}(x_1, t, s) = (2\pi)^{-1/2} \int dp_1 \sqrt{\left(\frac{m_0}{p_0}\right)} v(p_1, s)\bar{f}^*(p_1, s) \exp(-ipx) \tag{5.12}$$

In the case of the free K–G field we may similarly define the anti-particle amplitude $\bar{\varphi}^{(+)}(x_1, t)$ by the time-reversed particle amplitude $\varphi^{(+)*}(x_1, -t)$, thus obtaining the result

$$\bar{g}(p_1) = g^*(-p_1) \tag{5.13}$$

As for the D field, this equality may be attributed to both the incoming or outgoing evolutions, corresponding to the elastic particle–anti-particle collision process. We are now able to evaluate the time imprecisions of the D and K–G fields. The averages so obtained will be evaluated in respect to the particle amplitudes.

First, there exists the imprecisions of the time measurements performed on the free evoluting particle and anti-particle. These imprecisions are given, in agreement with methods previously used (Papp, 1973b), by the imaginary parts of the corresponding time-operator averages in respect to the single particle and single anti-particle amplitudes, respectively. Thus

$$\delta\tau^{(s)} \equiv \text{Im}\,\langle +, s|T_1^{(s)}(t)|+, s\rangle = \left\langle \frac{m_0^2}{2p_1^2 p_0} \right\rangle_{(s)} \tag{5.14}$$

for the D field and

$$\delta\tau \equiv \text{Im}\,\langle +|T_1\;\;(t)|+\rangle = \left\langle \frac{m_0^2}{2p_1^2 p_0} \right\rangle \tag{5.15}$$

for the K–G field, where the particle amplitudes have been normalised to unity. The time imprecisions of the particle and anti-particle states are identical and also possess the same form in D and K–G fields.

Evaluating the real part of the time-operator average in respect to state (5.1) one obtains

$$\text{Re}\,\langle +, -, s|\mathscr{T}_1^{(s)}(t)|+, -, s\rangle = -2\left\langle \frac{p_0}{p_1} \frac{\partial}{\partial p_1} \arg f(p_1, s) \right\rangle_{(s)} \tag{5.16}$$

This result means that the free evoluting particle–anti-particle pair could be formally considered as an object of the time-measurement, with the imprecision given by

$$-\operatorname{Im}\langle +,-,s|\mathscr{T}_1^{(s)}(t)|+,-,s\rangle = 2\left\langle \frac{m_0^2}{2p_1^2 p_0} \right\rangle_{(s)} \tag{5.17}$$

But the form of relations (5.16) and (5.17), and especially the presence of factor 2, recognise that the objects of the time-measurement are in fact the free evoluting particle and, respectively, the free evoluting anti-particle. A similar discussion is also valid for the K–G field.

Calculating the matrix element of the $\tilde{I}_1^{(s)}$ operator between the particle–anti-particle state $|+,-,s\rangle$ and the vacuum state $|0\rangle$, one obtains, using expression (5.8), the result

$$\tilde{\beta}_1^{(s)} \equiv -i\langle +,-,s|\tilde{I}_1^{(s)}|0 = \left\langle \frac{m_0}{2p_1 p_0} \right\rangle_{(s)} \tag{5.18}$$

The above average does not depend on the phase of the $f(p_1, s)$ amplitude or on the time parameter t. In these conditions the existence of an interaction supporting the pair annihilation process may be implicitly assumed. We are thus able to interpret the average $\langle m_0/2p_1p_0\rangle_{(s)}$ as the imprecision of the time measurements due to the virtual existence of the pairs annihilation process. Such a process arises, for example, in the fourth order approximation of the elastic particle–anti-particle collision-process. Consequently, the above imprecision becomes operative at the threshold energy (velocity) of the inelastic particle–anti-particle collision process. In this respect the threshold velocity has necessarily, to be attributed to the pair annihilation process and also to the subsequent production process of other particles. The evaluation of such a threshold velocity may be obtained in the usual manner, attributing to the constant $\hbar/2m_0c^2$ the role and meaning of the natural time unit in respect to the imprecision $\langle m_0/2p_1p_0\rangle_{(s)}$.

Commencing with the inequality

$$\left\langle \frac{m_0}{2p_1 p_0} \right\rangle_{(s)} \geqslant \frac{1}{2m_0} \tag{5.19}$$

we may conclude that the threshold velocity investigated is identical with the previously encountered velocity $v_{(1)} \equiv [\sqrt{(5)} - 1]/2\,c$. Attributing, by virtue of point (f) in the previous paper (Papp, 1973a), the imprecision meaning also to the expressions $\frac{3}{2}\tilde{\beta}_1^{(s)}$ and $2\tilde{\beta}_1^{(s)}$, one obtains

$$\frac{3}{2}\left\langle \frac{m_0}{2p_1 p_0} \right\rangle_{(s)} \geqslant \frac{1}{2m_0}, \qquad \langle v_1\rangle \leqslant v'_{(1)} \equiv \frac{\sqrt{(10)} - 1}{3} c \simeq 0{\cdot}721c \tag{5.20}$$

$$2\left\langle \frac{m_0}{2p_1 p_0} \right\rangle \geqslant \frac{1}{2m_0}, \qquad \langle v_1\rangle \leqslant v''_{(1)} \equiv \frac{\sqrt{(17)} - 1}{4} c \simeq 0{\cdot}781c \tag{5.21}$$

where the velocity on the right expresses the upper velocity for which the inequality on the left is fulfilled. One would expect the threshold velocities $v'_{(1)}$ and $v''_{(1)}$ so obtained to describe some deep inelastic peculiarities of the particle-anti-particle collision process.

The annihilation time imprecision of the K–G field may be similarly defined as for the D field, thus obtaining the result

$$\tilde{\beta}_1 = -i\langle +, -|\tilde{I}_1| 0 = \left\langle \frac{1}{2p_0} \right\rangle \tag{5.22}$$

Contrary to the D field, only the imprecisions $\frac{3}{2}\tilde{\beta}_1$ and $2\tilde{\beta}_1$ are able to define a threshold behaviour:

$$\frac{3}{2}\left\langle \frac{1}{2p_0} \right\rangle \geqslant \frac{1}{2m_0}, \qquad \langle v_1 \rangle \leqslant \frac{\sqrt{(5)}}{3} c \simeq 0{\cdot}745c \tag{5.23}$$

$$2\left\langle \frac{1}{2p_0} \right\rangle \geqslant \frac{1}{2m_0}, \qquad \langle v_1 \rangle \leqslant \frac{\sqrt{(3)}}{2} c \simeq 0{\cdot}866c \tag{5.24}$$

The 'annihilation' time imprecision $\langle 1/2p_0 \rangle$ so obtained expresses, besides the imprecision $\langle m_0{}^2/2p_1{}^2p_0 \rangle$, a contribution at total time imprecision. Consequently, for the K–G field, the total time imprecision contains the time-imprecision contribution of the free-evolution process and also the time-imprecision contribution arising from the virtual existence of the pairs annihilation processes. In this respect a well-defined meaning may be attributed to the total time imprecision.

The imprecisions of the proper-time measurements implied by the pairs annihilation processes may be similarly obtained. Indeed,

$$\tilde{\tilde{\beta}}_1^{(s)} = \left\langle \frac{p_1}{2p_0{}^2} \right\rangle_{(s)} \tag{5.25}$$

for the one-dimensional Dirac field and

$$\tilde{\tilde{\beta}}_j = \frac{j-1}{2m_0}, \qquad j = 1, 2, 3 \tag{5.26}$$

for the j-dimensional K–G field.

Actually only imprecision $2\langle p_1/2p_0{}^2 \rangle_{(s)}$ implies the existence of a threshold velocity:

$$2\left\langle \frac{p_1}{2p_0{}^2} \right\rangle_{(s)} \geqslant \frac{1}{2m_0}, \qquad \langle v_1 \rangle \leqslant v_{\tilde{s}} \equiv \frac{\sqrt{(2)}}{2} c \tag{5.27}$$

whose meaning will also be subsequently analysed.

6. *The Resonance-Emission Approximation of High-Energy Particle Production Processes*

We shall now prove that the virtual photons assumed to appear as a result of the particle–anti-particle annihilation processes and are able to support—at least qualitatively—the existence of certain resonance emission processes. Indeed, for the D-particle–anti-particle annihilation process, a 'hard' photonic emission should assume to appear starting with the threshold velocity $v_{(1)}$. On the other hand, at that threshold velocity, the annihilation imprecision $\langle m_0/2p_1p_0\rangle_{(s)}$ fulfils its role as a natural unit. As a consequence (Papp, 1973a) this time imprecision becomes a unit respecting the interaction time-shift.

$$\frac{d}{dp_0}\,\delta(p_1) = N\frac{m_0}{2p_1p_0} \tag{6.1}$$

where N is a parameter. Consequently

$$\delta(p_1) = \frac{N}{2}\operatorname{arctg}\frac{p_1}{m_0} \tag{6.2}$$

where the initial condition $\delta(0) = 0$ has been considered. Performing the high-energy approximation

$$\operatorname{tg}\delta(p_1) \simeq \frac{N\pi}{4} - \frac{Nm_0}{p_1} + \dots \tag{6.3}$$

one obtains a Breit–Wigner resonance, expressed by the pole

$$p^{(c)} = \frac{2N^2\pi m_0}{\pi^2N^2 + 16} - i\frac{8Nm_0}{\pi^2N^2 + 16} \tag{6.4}$$

which produces in the complex energy plane, a resonance having, e.g. for $N = 4$, approximately the energy $p_0^{(r)} \simeq 1{\cdot}3m_0$ and the width $\Gamma/2 \simeq 0{\cdot}1m_0$. Similarly, considering the K–G time imprecision $\langle 1/2p_0\rangle$ as a unit in respect of the interaction time-shift, one obtains the phase-shift evaluation

$$\delta(p_1) = \frac{N}{2}\ln\frac{p_0}{m_0} \tag{6.5}$$

Performing the previously used high-energy approximation of the logarithmic function, results in the existence of a resonance given by the complex energy pole

$$p_0^{(r)} - i\frac{\Gamma}{2} = \frac{2N^2}{N^2+4}m_0 - i\frac{4N}{N^2+4}m_0 \tag{6.6}$$

Taking $N = 4$ one obtains $p_0^{(r)} \simeq 1{\cdot}6m_0$ and $\Gamma/2 \simeq 0{\cdot}8m_0$.

The high-energy behaviour implied by the D-annihilation proper-time imprecision $\langle p_1/2p_0^2\rangle_{(s)}$ is, within the same approximation, identical with the one above. Indeed, the imprecision $\langle p_1/2p_0^2\rangle_{(s)}$ expresses the standard form of the space imprecision corresponding to the time imprecision $\langle 1/2p_0\rangle_{(s)}$. Consequently the imprecision $\langle p_1/2p_0^2\rangle_{(s)}$ has to fulfil the role of the space unit, thus obtaining the phase-shift evaluation (6.5). In agreement with the above results, we may consider that the possibility of the annihilation and of the subsequent production processes and also the threshold behaviour may be qualitatively stated. We must also mention that besides the existence of the resonance emission the possibility of the so-called radiative coherent emission (Horn & Silver, 1970) has to be taken into consideration.

In these conditions some explanation may be given to the high-energy annihilation-production processes $A^+ + A^- \to B^+ + B^-$ + other particles or $A^+ + A^- \to 2B$ + other particles. Using energy conservation law we may express the threshold behaviour of the above processes, defining the upper value of the rest-mass of the produced B-particle $(m_0^{(B^\pm)} = m_0^{(B)})$ by the threshold energy of the incoming A-particle. In this sense the rest-mass upper bounds, corresponding to the threshold velocities $v_{(1)}$, $v'_{(1)}$ and $v''_{(1)}$, are given by

$$m_0^{(B)} \leqslant p_0(v_{(1)}) \simeq 1{\cdot}272 m_0 \tag{6.7}$$

$$m_0^{(B')} \leqslant p_0(v'_{(1)}) \simeq 1{\cdot}443 m_0 \tag{6.8}$$

and

$$m_0^{(B'')} \leqslant p_0(v''_{(1)}) \simeq 1{\cdot}600 m_0 \tag{6.9}$$

respectively. As a consequence, the possibility of hadronic production processes, e.g.

$$m_0^{(B)} < 1191 \text{ MeV}, \qquad p + \bar{p} \to 2\Sigma_0, \qquad \Sigma^+ + \Sigma^- \tag{6.10}$$

$$m_0^{(B')} < 1354 \text{ MeV}, \qquad p + \bar{p} \to 2\Xi_0, \qquad 2\Sigma_0,\ \Sigma^+ + \Sigma^- \tag{6.11}$$

$$m_0^{(B'')} < 1501 \text{ MeV}, \qquad p + \bar{p} \to 2\Xi_0, \qquad 2\Sigma_0,\ \Sigma^+ + \Sigma^- \tag{6.12}$$

is explained in this way also. On the right are the inelastic reactions which are compatible with the upper rest-mass calculated on the left. The presence of other particles has been neglected.

Significant results referring to the high-energy behaviour of the colliding K–G particles may also be obtained. The upper rest-mass bounds corresponding to the threshold velocities $[\sqrt{(5)}/3]c$ and $[\sqrt{(3)}/2]c$ are given by $1{\cdot}5 m_0$ and $2m_0$ respectively. In this way is explained the threshold behaviour of the high-energy pionic production reactions

$$\pi^+ + \pi^- \to \pi^+ + \pi^- + \pi_0, \qquad 3\pi_0 \tag{6.13}$$

and

$$\pi^+ + \pi^- \to \pi^+ + \pi^- + \pi^+ + \pi^-, \qquad \pi^+ + \pi^- + 2\pi_0, \qquad 4\pi_0 \tag{6.14}$$

respectively. Other similar cases may also be considered.

Further developments of the above interpretations may be obtained, performing a more exact and complete systemisation of the experimental data and using more refined and improved approximations.

7. *Quanto-Mechanical Peculiarities of the Coulombian Interaction*

Under the conditions of which the existence of a charged particle and antiparticle is considered, the presence of the electromagnetic interaction and particularly the presence of the coulombian interaction is implied. Proving that certain aspects of the threshold behaviour implied by the imprecision description of the potential energy of the coulombian interaction are related with the threshold behaviour of the previously analysed time annihilation imprecisions, it may be assumed that certain responsibilities of the high-energy behaviour of the annihilation processes may be attributed to the 'coulombian' interaction.

For this purpose we shall consider the collision system of the electrostatically interacting particle-anti-particle system. Generally the attractive coulombian interaction of two charged particles of the same rest-mass may be described within the same framework. We shall consider for convenience the wave-packet description of a K–G particle-anti-particle pair. The calculations are similar for the D particles. The plane-wave product corresponding to the free moving particles takes in the centre-of-mass system (Blohinzew, 1970) the form:

$$\exp i(\mathbf{p}_1 \cdot \mathbf{r}_2 - p_0^{(1)}t) \exp i(\mathbf{p}_2 \cdot \mathbf{r}_2 - p_0^{(2)}t) = \exp i(\tilde{\mathbf{p}} \cdot \mathbf{r} - \tilde{p}_0 t) \qquad (7.1)$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\mathbf{p}$ is the centre-of-mass momentum and $\tilde{\mathbf{p}}_0 = 2(\mathbf{p}^2 + m_0{}^2)^{1/2}$. With the coulombian interaction being a long-range interaction the incoming and outgoing states describing the elastic particle-anti-particle collision process are only approximately free states. In order to state the free evolution, the coulombian interaction potential e^2/r has to be replaced, for example by $e^2/r \exp(-\epsilon r)$, where $\epsilon \to 0$. We may now suppose that at large t-values the evolution of the collision system is described, in the first approximation order, by the free K–G wave packet

$$\varphi(\mathbf{r}, t) = (2\pi)^{-3/2} \int \frac{d\mathbf{p}}{\sqrt{(2\tilde{p}_0)}}\, a(\mathbf{p}) \exp i(\mathbf{p} \cdot \mathbf{r} - \tilde{p}_0 t) \qquad (7.2)$$

In this approximation, the average value of the potential energy may be defined as

$$\left\langle \frac{e^2}{r} \right\rangle = e^2 \left(\varphi(\mathbf{r}, t), i\overleftrightarrow{\partial}_t \frac{1}{r} \varphi(\mathbf{r}, t) \right) \qquad (7.3)$$

where $r = |\mathbf{r}|$, $t \gg 0$, and where the wave packet has been normalised to unity. It may be stated that within the above conditions there is no possibility of producing an adequate imprecision description of the potential energy. We shall bypass this difficulty which requires certain mathematical conditions to guarantee the possibility of solving the imprecision evaluation problem and especially the evaluation problem of the binary potential energy operator in the momentum representation.

In this respect let us consider that the form factor $a(\mathbf{p})$ takes the particular form

$$a(\mathbf{p}) = \frac{1}{p}\, a(p) f(\theta, \varphi) \qquad (7.4)$$

where

$$f(\theta, \varphi) = \begin{cases} 1, & \theta, \varphi \in \mathscr{D}_{\theta,\varphi} \\ 0, & \theta, \varphi \notin \mathscr{D}_{\theta,\varphi} \end{cases} \tag{7.5}$$

and where $\mathscr{D}_{\theta,\varphi}$ expresses a certain domain defined by

$$\mathscr{D}_{\theta,\varphi} = \left\{ \theta, \varphi \left| \frac{\pi - \delta\theta}{2} \leqslant \theta \leqslant \frac{\pi + \delta\theta}{2},\ 0 \leqslant \varphi \leqslant \delta\varphi \right. \right\} \tag{7.6}$$

The angular parameters $\delta\theta$ and $\delta\varphi$ will be chosen to take sufficiently small—but non-zero—and equal values. Using the first-order $\delta\theta$-approximation it may easily be proved that the average of the momentum components with respect to state (7.1) are

$$\langle p_1 \rangle = \langle p \rangle, \qquad \langle p_2 \rangle = \frac{\delta\varphi}{2} \langle p \rangle \approx 0, \qquad \langle p_3 \rangle = 0 \tag{7.7}$$

so that, as one would expect, the form-factor (7.4) has to be associated with the collision products scattered along the Ox_1-axis. (In this way the one-dimensionality condition which is needed to perform the binary time description and also the proper-time description of the D field has been, at least approximately, reproduced). As the wave-packet form-factor is influenced by the measuring apparatus, a certain experimental situation has been established by virtue of the particular form of expression (7.4). The measuring apparatus, which is intended to record the charged particles scattered outside the momentum-space domain $\mathscr{D}_{\theta,\varphi}$, become inoperative. Consequently the observable of the potential energy which is able to support the above experimental situation will be effectively chosen as

$$V^{\text{eff}}(r, \theta, \varphi) = \frac{\tilde{e}^2}{r} \tilde{f}(\theta, \varphi) \exp(-\epsilon r) \tag{7.8}$$

where $\tilde{e}$ expresses the effective charge parameter. The function $\tilde{f}(\theta, \varphi)$ is similarly defined in relation (7.5), but actually

$$\tilde{\mathscr{D}}_{\theta,\varphi} = \left\{ \theta, \varphi \left| \frac{\pi - \tilde{\delta}\theta}{2} \leqslant \theta \leqslant \frac{\pi + \tilde{\delta}\theta}{2},\ 0 \leqslant \varphi \leqslant \varphi_0 \right. \right\} \tag{7.9}$$

where $2\pi \geqslant \varphi_0 \gg \tilde{\delta}\theta$ and where the angular parameter $\tilde{\delta}\theta$ takes vanishingly small values.

Performing the calculations in the first-order $\delta\theta$- and $\tilde{\delta}\theta$-approximations, it may be proved that

$$\int d\mathbf{r} \frac{\tilde{f}(\theta, \varphi)}{r} \exp i(\mathbf{p} - \mathbf{p}') \cdot \mathbf{r} = \frac{4\varphi_0}{\delta\theta} \frac{1}{(p - p')} P \frac{1}{p - p'} \tag{7.10}$$

where the modulus $|\mathbf{p} - \mathbf{p}'|$ has been replaced by the difference $(p - p')$. In these conditions we finally obtain the evaluation

$$\left\langle V^{\mathrm{eff}}(r, \theta, \varphi)\right\rangle = n\left(\tilde{e}\frac{\delta\theta}{2\pi}\right)^2\left\langle -ip - ip^2\frac{d}{dp}\right\rangle \tag{7.11}$$

where $n \equiv \varphi_0/\delta\theta \gg 1$ and where the average on the right is performed in the p-momentum representation. The relations

$$\lim_{t\to+\infty} P\frac{1}{p - p'}\exp i(\tilde{p}_0' - \tilde{p}_0)t = -i\pi\,\delta(p - p') \tag{7.12}$$

$$\frac{d}{dp}\delta(p - p') = -2\delta(p - p')P\frac{1}{p - p'} \tag{7.13}$$

and

$$\int_0^\infty e^{-\epsilon r}\sin(p - p')r = P\frac{1}{p - p'} \tag{7.14}$$

have been used. We shall now establish the value of the effective charge requiring that

$$\tilde{e} = \sqrt{\left(\frac{1}{n}\right)}\frac{2\pi}{\delta\theta}e \tag{7.15}$$

Generally the meaning of the above relation has to be analysed in terms of the corresponding $\tilde{e}$- and e-charge distribution functions. On the other hand, by means of relation (7.15), the existence of a 'discrete' space described by the parameter n and also by the momentum-space imprecision $\delta\theta$ is effectively introduced into the collision description formalism. It will be subsequently proved that the above choice is the suitable one.

We are now able to define the effective potential energy operator in the p-momentum representation as

$$\hat{u} = -ie^2\left(p + p^2\frac{d}{dp}\right) \tag{7.16}$$

Applying the space-time imprecision evaluation methods and averaging for this purpose the operator $\hat{u}$ with respect to the energy representation state $[\sqrt{(p_0/p)}]a(p)$, it may be easily proved that the electrostatic potential-energy imprecision is

$$\delta^{(1)}u = m_0{}^2e^2\left\langle\frac{p}{2p_0^2}\right\rangle \tag{7.17}$$

In agreement with binary description formalism we may also choose, for convenience, the expression $\delta u = 2\delta^{(1)}u$ as the operative imprecision. On the other hand if we assume, as usual, that the form factor $a(p)$ takes appreciable values only in a relatively small vicinity of the $\langle p\rangle$-average, the average (7.11) takes approximately the form

$$\langle \hat{u}\rangle \simeq \frac{e^2}{4\delta\tilde{s}}\frac{1}{\delta\tilde{s}}\left\langle \frac{d}{dp}\arg a(p)\right\rangle \equiv N(\langle v\rangle)\frac{e^2}{4\delta\tilde{s}} \tag{7.18}$$

where $\delta\tilde{s} = \langle 1/2p\rangle$ expresses the total space imprecision and the function $N(\langle v\rangle)$ the multiplicity of the collision space-shift $\langle (d/dp)\arg a(p)\rangle$ with respect to the total space imprecision. As the total space imprecision actually possesses the meaning of the effective space unit needed to 'measure' the interaction space-shift, it appears possible to attribute the role of the electrostatic energy unit to the expression

$$\delta\tilde{u} = \frac{e^2}{4\delta\tilde{s}} \simeq \tfrac{1}{2}m_0 e^2\langle v\rangle(1-\langle v\rangle^2)^{-1/2} \tag{7.19}$$

The necessary conditions needed to assure the measurable meaning of the space-shift and of the potential-energy average (7.18) are given by $N(\langle v\rangle) > 1$ and

$$N(\langle v\rangle)\frac{e^2}{4\delta\tilde{s}} > e^2 m_0^2\left\langle\frac{p}{p_0^2}\right\rangle \tag{7.20}$$

respectively, so that both conditions are fulfilled only when $\langle v\rangle < [\sqrt{(2)}/2]c$ (see also Jaffe & Shapiro, 1972). Consequently the coulombian interaction loses its observable meaning for velocities larger than the threshold velocity $[\sqrt{(2)}/2]c$, also when $\delta\tilde{s} < \delta^c s$. This threshold velocity also expresses the velocity value for which the δu-imprecision and the $\delta\tilde{u}$-unit take the common value

$$\delta^c u = \frac{e^2}{4\delta^c s} = m_0 e^2\frac{c}{2\hbar} \tag{7.21}$$

where $\delta^c s$ expresses the natural space unit. In connection with the above results, we are able to state expression (7.21) as the natural unit of electrostatic potential energy. Requiring the $\langle\hat{u}\rangle$–average to be larger than $4\delta^{(1)}u$, it results that the coulombian energy possesses a well-defined measurable meaning when $\langle v\rangle \leqslant (\sqrt{3}/2)c$.

In the previously formulated interpretations we should have to consider that the natural unit expresses the lower δu-imprecision value up to which–as a result of the appearance of the inelastic effects–the colliding products maintain their initial individuality. In this respect it has been implicitly assumed that a low-energy domain of the form $(0, v^{(tr)} < c)$ exists in which the imprecision is larger than the natural unit. But actually $\delta^c u > \delta u$, when $\langle v\rangle < [\sqrt{(2)}/2]c$ and $\langle v\rangle > [\sqrt{(2)}/2]c$ respectively, so that additional interpretations are needed. Indeed, up to velocity $[\sqrt{(2)}/2]c$ the coulombian interaction maintains its initial observable significance. This fact means that the low-energy annihilation process $A^+ + A^- \to 2\gamma$ is only apparently 'inelastic', because there also exist the possibility of the pair-production process $2\gamma \to A^{+} + A^{-}$. The existence of competition between the above processes may also be assumed. On the other hand. for velocities larger than $[\sqrt{(2)}/2]c$, properly inelastic effects arise. In this last case the inelastically emitted photons–which have to change the initial significance of the coulombian interaction–are able to support new particle production processes. Consequently, up to the threshold velocity $[\sqrt{(2)}/2]c$, one would expect the existence of a virtual or 'elastic' photonic emission process, whereas the properly inelastic photons have to appear only for velocities larger than $[\sqrt{(2)}/2]c$, when the role of the δu-imprecision becomes operative.

In such a situation–and generally in similar conditions–the observable meaning of the coulombian interaction has to be directly expressed in respect of the natural unit $\delta^c u$. Indeed, considering that the electrostatic potential energy is able to possess a measurable meaning only when

$$N(\langle v \rangle)\frac{e^2}{4\delta\tilde{s}} > \frac{e^2}{4\delta^c s} \tag{7.22}$$

where $N(\langle v \rangle) > 1$, one reobtains the threshold velocity $[\sqrt{(2)}/2]c$. Thus, the consistency of the above interpretations is proved. It may also be verified that the threshold velocity $[\sqrt{(2)}/2]c$ does not depend on the choice of discrete-space parameters n and $\delta\theta$.

One can now easily remark that the threshold velocities $(\sqrt{3}/2)c$ and $(\sqrt{2}/2)c$ are identical to the ones of (5.24) and (5.27) respectively. It may easily be proved–neglecting the threshold velocity $v_{(1)} = [\sqrt{(5)} - 1/2]c$–that the values taken by the $\delta\tilde{u}$-unit (and the δu-imprecision) for the remaining previously calculated threshold velocities are relatively adjacent, and may be mutually covered within extended binary equivalence. In these conditions we may conclude that a mutual connection between the threshold behaviour of the pairs annihilation processes and the threshold peculiarities of the (effective) coulombian interaction may at least be qualitatively established. We should also state that the threshold velocities defined in Section 5 are not operative with respect to the electrons. Indeed, there are no (known) particles whose rest-mass value is included in the interval $(m_0, 2m_0]$, where m_0 is the rest-mass of the electron. In such conditions, a further suitable threshold velocity, taking sufficiently large values, has yet to be defined. For this purpose, as we shall prove in the following section, the existence of a separate natural unit of space will be introduced.

8. *The Natural Electromagnetic Space Unit*

The effective unit $\delta\tilde{u}$ mathematically, is, an unbounded increasing function of the velocity. For this purpose certain limitations may be imposed on the unit requiring that

$$4\delta\tilde{u} < m_0 c^2 \tag{8.1}$$

In this respect, we mention that an effective unit becomes physically meaningless when it takes too large values. With the entity $\delta\tilde{u}$ also being generally an energy unit, the above inequality is needed in order to preserve the possibility of performing energy measurements.

As a consequence

$$\delta\tilde{s} > \frac{e^2}{m_0 c^2} \tag{8.2}$$

so that a new lower bound has to be imposed on the total-space imprecision. In this way the classical electromagnetic particle radius

$$\delta^{(em)}s = \frac{e^2}{m_0 c^2} = \frac{e^2}{\hbar c}\,\delta^c s \tag{8.3}$$

may be defined, in terms of the quanto-mechanical imprecision description, as the natural electromagnetic space unit. Setting $\delta\tilde{s} = \delta^{(em)}s$, results in the existence of an 'electromagnetic' energy-threshold given by

$$p_0^{(em)} = m_0 c^2 \left(1 + \frac{\hbar^2 c^2}{e^4}\right)^{1/2} \simeq 137 m_0 c^2 \tag{8.4}$$

and also the existence of a new threshold velocity given by

$$v^{(em)} = \left(1 + \frac{e^4}{\hbar^2 c^2}\right)^{-1/2} c \simeq 0{\cdot}999974c \tag{8.5}$$

which possesses the order of magnitude needed to explain qualitatively the deep inelastic electron-positron production processes. The imprecisions $\frac{3}{2}\delta\tilde{s}$ and $2\delta\tilde{s}$ imply approximately the energy thresholds $225 m_0 c^2$ and $274 m_0 c^2$ respectively. This latter threshold energy allows the existence of the extreme high-energy inclusive reaction $e^+ + e^- \to$ pions, which has recently been taken as evidence (see e.g. Bacci *et al.*, 1972). It may also be remarked that the natural electromagnetic space unit is approximately equal to the double value of the pionic natural space unit $\hbar/m_0^{(\pi)}c$. By virtue of this coincidence one may assume, at sufficiently high energies, the possibility of certain coexistence effects between the electromagnetic and the hadronic structure of the matter. It may now be proved, in agreement with previous results, that such an assumption is physically meaningful. Indeed at sufficiently high energies the coulombian interaction loses its observable meaning and is, in fact, substituted by other interactions, particularly by the strong interaction. In this sense we have to understand the previously formulated assumption that only certain responsibilities of high-energy particle production processes may be attributed to the 'coulombian' interaction.

We may thus conclude that, in order to explain high-energy annihilation processes, and especially the extreme high-energy electron-positron annihilation process, the existence of the natural electromagnetic space unit is needed. In these conditions, as a consequence of the fact that $\delta^{(em)}s \simeq \frac{1}{137}\delta^c s$, we find the existence of a relatively large threshold-energy 'gap' between the threshold energy $2m_0$ (which is the largest threshold energy calculated in Section 5) and the threshold energies calculated above.

For instance, the existence of the gravitational interaction has been neglected. The imprecision description of the gravitational interaction is, up to the coupling constant, identical to that of the coulombian interaction. Consequently, in order to obtain the imprecision description of the gravitational interaction, the fine-structure constant $e^2/\hbar c$ has to be substituted by the gravitational coupling constant $g(m_0^2/\hbar c)$. The value of this latter coupling constant is practically negligible with respect to the fine-structure constant. In fact the gravitational interaction loses its observable meaning at the same threshold velocity of $[\sqrt{(2)}/2]c$. The 'gravitational' space unit is given by $g(m_0/2c^2)$, so that the 'gravitational' threshold velocity takes the form

$$v^{(\mathrm{grav})} = \left(1 + \frac{g^2 m_0^4}{\hbar^2 c^2}\right)^{-1/2} c$$

which is practically identical to c.

9. *Conclusions*

Using imprecision description methods we have analysed some qualitative and quantitative aspects of the threshold behaviour implied by the existence of the time- and proper-time imprecisions associated with the D and K–G pairs annihilation (and subsequent) production processes. It is significant that threshold behaviour is able to be supported, at least qualitatively, by the imprecisions description of the elementary coulombian and gravitational interactions. Thus the imprecision description formalism is able to elucidate in a relatively simple manner profound peculiarities of the high-energy behaviour of the collision processes. We are also able to consider that the essential physical content of the imprecision description formalism consists of explaining the role and meaning of the implied threshold energies or threshold velocities.

Qualitative and quantitative distinctions between the proper-time and time descriptions of the D and K–G fields have been taken as evidence. Thus the proper time is an observable which may be defined irrespective of the rest-state or motion-state of the reference frame, whereas the time is an observable which may be defined, along an arbitrary direction, only with respect to moving particles. On the other hand, the imprecision associated with annihilation processes is sensitive in respect of statistics, and the time- and proper-time measurements respectively.

It has been proved that the extension of the binary description formalism to the 'matter', and especially to the coulombian and gravitational interactions, require additional mathematical conditions which effectively express the existence of a discrete space. In this way, in agreement with opinions expressed by Cole (1972), certain modifications of the actual concept of the electrostatic potential energy are implied. Finally we must mention that interpretations formulated throughout this paper, which are rather qualitative, are liable to amendment and agreement with a more complete and systematic set of experimental data required.

References

Bacci, C. *et al.* (1972). Preprint, Frascati.
Bjorken, J. D. and Drell, S. D. (1965). *Relativistic Quantum Fields.* McGraw-Hill, New York.
Blohinzew, D. I. (1970). *Prostronstvo i vremia v mikromire, Izd.* Nauka, Moskwa.
Bohm, D., Hilley, B. J. and Stewart, A. E. G. (1970). *International Journal of Theoretical Physics,* Vol. 3, No. 3, p. 171.
Cole, E. A. B. (1972). *International Journal of Theoretical Physics,* Vol. 1, No. 1, p. 3.
Horn, D. and Silver, R. (1970). Preprint, TH 1226-CERN
Jaffe, J. and Shapiro, I. I. (1972). *Physical Review,* **6D**, 405.
Papp, E. (1972). *Nuovo Cimento,* **10B**, 471.
Papp, E. (1973a). *International Journal of Theoretical Physics,* Vol. 8, No. 6, p. 429.
Papp, E. (1973b). *International Journal of Theoretical Physics,* Vol. 9, No. 1, p. 101.
Vialtzew, A. N. (1965). *Diskretnoe prostronstvo i vremia, Izd.* Nauka, Moskwa.